

REAL FREQUENCY TECHNIQUE APPLIED TO SYNTHESIS OF LUMPED  
BROADBAND MATCHING NETWORKS WITH ARBITRARY NONUNIFORM LOSSES FOR MMIC'S

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ABSTRACT

A new computer-aided synthesis technique is presented in this paper for treating the synthesis of lumped matching networks with arbitrary nonuniform losses. It is especially applicable to the design of the broadband amplifiers in MMIC's. A new useful theorem and corollary are developed for the transformation between lossy or lossless network and lossless one, so that the design of the lossy matching networks is quite simplified and it can yield any complex models of the lumped elements with arbitrary nonuniform losses than that of Ref.(1). An example is given to show the general applications of the new method in the monolithic broadband amplifier design.

INTRODUCTION

With the rapid development of GaAs material, resolution of many troublesome device reliabilities and wafer processing in the last few years, more and more MMIC's became practical. In the design of MMIC's, a matching network is usually composed of lumped reactive elements and/or transmission-line elements. Lumped elements are often essential since they not only occupy less GaAs chip area, but also have broader bandwidth capability. Unfortunately, the lumped elements fabricated on the semi-insulating GaAs substrates have losses which are too large to be neglected. Due to the lack of a lossy synthesis technique, most circuit designers often synthesize a lossless matching network first and then simply add loss elements to each lumped circuit components. Since the loss elements added have different effects on the transduce power gain (TPG) at different frequencies, the gain response will be out of control. Developing a lumped lossy matching network synthesis technique is therefore very desirable for MMIC's.

The exact synthesis of the lumped networks with lossy elements has been an open problem with long standing for the general case of arbitrary nonuniform dissipation. In 1939, Darlington in his well-known paper (2) first used the reactive elements which have losses of the semi-uniform type (i.e., all inductors have one quality factor  $Q$ , all capacitors have another  $Q$ ) to synthesize a lossy filter. After that, Andersen (3) developed Darlington's work and provided realizability conditions for the semi-uniform type, three synthesis procedures and insertion loss theory. Based on Andersen's theory, Su (4) extended the analytical theory of the broadband matching network (5,6,7) to the semi-uniform cases. Though this analytical theory has its theoretical value, it is, however, too complicated to be applied to the practical

design of MMIC's. In 1984, Liu (1), using both analytical and CAS technique, solved the lossy synthesis problem for the case of unequal inductor and capacitor losses (i.e., semi-uniform losses) with arbitrary circuit topology and realizable gain functions. Even though this new result is directly applicable to the synthesis of the matching networks for MMIC's, the new theory did not take the parasitic reactances of the lossy circuit elements into account and only used the simplified FET model, as well as could not synthesize the circuit topology directly!

In order to solve the problem mentioned above, we provide a new method which can directly synthesize the lumped arbitrary nonuniform lossy matching networks for MMIC's. Since the simplified "Real Frequency" technique (SRFT)(8) is used to synthesize a primary lossless double matching network, our method has all the advantages the SRFT has. With the help of transformation provided by our theorem, the lossless reactive lumped elements can be replaced by the lumped circuit elements with arbitrary nonuniform losses including skin, conductor and dielectric losses. Therefore, all kinds of losses in the matching network are considered except the possible derivations of the lossy elements models from the actual ones. If it is necessary to consider the more exact models of the lossy elements to achieve even better performance, a CAD program may be used to modify the elements values of the actual lossy L's and C's of the matching network. In conclusion, it can be predicted that the new method we developed for the synthesis of the lossy matching networks for MMIC's will have a wide applications in the more general cases.

TRANSFORMATION BETWEEN  
LOSSY AND LOSSLESS MATCHING NETWORKS

A matching network is the most important part in the design of MMIC's. The classical matching problem is one of constructing a lossless matching network between a generator and a load, such that the transfer of power is maximized over a prescribed frequency band. But the modern matching problem, especially, the matching problem in MMIC's, is one of synthesizing and/or designing a lossy matching network. Thus, the usual approaches for the synthesis of the lossless matching network, such as the analytic gain-bandwidth theory (5,6,7), the commercially available CAD program AMPSYN (9), and the newly developed "Real Frequency" technique (10,8,11), can not be used to synthesize the lossy matching network, then it is necessary to find a new method which can synthesize

the lossy matching network directly!

Darlington, in his paper (2), had talked something about dissipative reactance network. He presented us a general theorem described by Bode which states that if each element of a network  $N$  produces an individual impedance proportional to  $Z_1$  or  $Z_2$ , any impedance of the complete network will be the product of  $Z_2$  and a rational function of  $Z_1/Z_2$ . In the Bode's theorem,  $Z_1$  and  $Z_2$  may be any physically realizable impedances. To our knowledge, although this important theorem was used by several authors (3, 4, 12, 13) to solve the problems of semi-uniform lossy networks, the general applications of it have not been found yet.

By careful consideration and derivation, it is found that any network, whether it is lossy or lossless one, can be transformed to a lossless network  $M$ , if it satisfies the condition of the Bode's statement.

Theorem: Suppose each element of a lossy or lossless network  $N$  produces an individual impedance proportional to  $Z_1$  or  $Z_2$ , any impedance of the network, if it exists, can be transformed to a corresponding impedance of the lossless network  $M$  by the following transformation.

$$\tilde{Z} = Z_1 \sqrt{Z_1 Z_2} = \tilde{Z}_c f(\tilde{Z}_L / \tilde{Z}_c) \quad (1a)$$

In reverse, the transformation is also true. That is,

$$Z = \sqrt{Z_1 Z_2} \quad \tilde{Z} = Z_2 f(Z_1 / Z_2) \quad (1b)$$

where  $Z$  is any impedance of the lossy or lossless network  $N$ ;

$\tilde{Z}$  is a corresponding impedance of the lossless network  $M$ ;

$\tilde{Z}_L = \Omega$ , is an impedance of equivalent inductor;

$\tilde{Z}_c = 1/\Omega$ , is an impedance of equivalent capacitor;

$\Omega = \sqrt{Z_1/Z_2}$  is an equivalent angular frequency; and  $f(x)$  is a rational function of  $x$ .

From the theorem, it is clear to see that the right hand side of the equation (1a) has the same form as that of the equation (1b). That is to say that all kinds of impedances  $Z_1$  and  $Z_2$ , can be transformed to the corresponding lossless ones, whether they possess the properties of the semi-uniform losses or frequency dependent losses! For example, If both skin and dielectric losses are considered, then all the impedances of the lossy circular inductors and MIM (metal-insulator-metal) capacitors (14) may be written as

$$Z_{Lj} = L_j (s + d_L \omega^{1/2}) = L_j Z_1 \quad (2a)$$

$$Z_{Cj} = \frac{1}{C_j} \left( \frac{1}{s} + \frac{\omega^{1/2}}{d_C} + \frac{1}{d_d \omega} \right) = \frac{1}{C_j} Z_2 = \frac{1}{C_j Y_2} \quad (2b)$$

where  $Z_{Lj}$  and  $Z_{Cj}$  are branch impedances corresponding to the lossy inductor and capacitor;

$L_j$  and  $C_j$  are corresponding inductance and capacitance;

$Z_1$  and  $Z_2 = 1/Y_2$  are the same as those in the theorem;

$s$  is a complex angular frequency; and

$d_L$  and  $d_C$  are inductor and capacitor losses or dissipation factors which can be written as

$$d_L = \omega_0^{1/2} Q_L, \quad (3a)$$

$$d_C = \omega_0^{1/2} Q_C, \quad (3b)$$

here,  $Q_L$  and  $Q_C$  are skin loss quality factors of the lossy inductor and capacitor, and  $\omega_0$  is an angular frequency at which the  $Q$ 's are defined.  $d_d$  is equal to the quality factor  $Q_d$ , which is determined by the dielectric loss of the GaAs material.

Thus, the corresponding lossless reactive elements have an equivalent angular frequency given by

$$\Omega = \sqrt{(s + d_L \omega^{1/2}) / (1/s + \omega^{1/2} d_C + 1/d_d \omega)} \quad (4)$$

Since the scattering parameters are often used in the design and analysis of the microwave circuits, the following corollary is very useful for obtaining the scattering parameters of the lossy matching network  $N$  from the ones of the lossless matching network  $M$ .

Corollary: If each element of a lossy or lossless network  $N$  produces an individual impedance proportional to  $Z_1$  or  $Z_2$ , there will always exist a transformation between the scattering matrix of the lossy or lossless network  $N$  and the one of the lossless network  $M$ .

$$\tilde{S}(\Omega) = \tilde{S}(\sqrt{Z_1/Z_2}) = \left[ (I + S) - \sqrt{Z_1 Z_2} (I - S) \right] \left[ (I + S) + \sqrt{Z_1 Z_2} (I - S) \right]^{-1} \quad (5a)$$

$$S(S) = S(Z_1, Z_2) = \left[ \sqrt{Z_1 Z_2} (I + S) - (I - S) \right] \left[ \sqrt{Z_1 Z_2} (I + S) + (I - S) \right]^{-1} \quad (5b)$$

where  $S(S)$  is any scattering matrix of the lossy or lossless network  $N$ ;

$\tilde{S}(\Omega)$  is a corresponding scattering matrix of the lossless network  $M$ ; and

$I$  is the identity matrix.

#### SIMPLIFIED "REAL FREQUENCY" TECHNIQUE APPLIED TO SYNTHESIS OF THE LUMPED MATCHING NETWORK WITH ARBITRARY NONUNIFORM LOSSES

Since the simplified "Real Frequency" technique (SRFT) (8) only employs the lossless scattering parameters to optimize the transduce power gain (TPG) of a lossless matching system with a complex generator and a load, and to realize the lossless matching networks, it is easier to obtain the scattering parameters of the lossy matching networks from the ones of the lossless matching networks by the transformation of the corollary and to use the lossy scattering parameters in the design of the broad-band amplifier with the lossy matching networks.

Algorithm: Computation of the TGP of the monolithic multistage FET amplifier.

First, suppose  $\tilde{e}_{11}(s)$ , which is a unit normalized input reflection factor of the lossless matching network  $M$ , is given by

$$\tilde{e}_{11}(s) = \frac{h(s)}{g(s)} = \frac{h_1 + h_2 s + h_3 s^2 + \dots + h_{n+1} s^n}{g_1 + g_2 s + g_3 s^2 + \dots + g_{n+1} s^n} \quad (6)$$

where  $n$  specifies the maximum number of reactive elements in  $M$ . Then, employing the well known Belevitch representation (15), the unit normalized scattering parameters of  $M$  are given by

$$\tilde{e}_{11}(s) = h(s)/g(s) \quad (7a)$$

$$\tilde{e}_{12}(s) = \tilde{e}_{21}(s) = (+/-) s^k / g(s) \quad (7b)$$

$$\tilde{e}_{22}(s) = -(-1)^k h(-s)/g(s) \quad (7c)$$

where for simplicity,  $M$  is assumed to be a minimum phase structure with transmission zeros only at  $s=j\infty$  and  $s=j0$ . This is a convenient assumption since it assures realization without coupled coils, except possibly for an impedance level transformer, which usually can be avoided by fixing  $h_1$  and  $h_{n+1}$  to be zeros for the lowpass and high-pass cases, and by using Norton transformation for the bandpass case.  $k \geq 0$  is an integer and specifies the order of the transmission zeros. Since the matching network is lossless, it follows that

$$g(s)g(-s) = h(s)h(-s) + (-1)^k s^{2k} \quad (8)$$

If the iterative approach is used to optimize the TPG, the coefficients of the numerator polynomial  $h(s)$  are chosen as unknowns. To construct the scattering parameters of  $M$ , it is sufficient to generate the Hurwitz denominator polynomial  $g(s)$  from  $h(s)$ . It can be readily shown that once the coefficients of  $h(s)$  are initialized at the start of the optimization process and the maximum complexity of the matching network  $M$  is specified (i.e.,  $n$  and  $k$  are fixed),  $g(s)$  can be generated as a Hurwitz polynomial by explicit factorization of the equation (8). Thus, the physical realisability of the scattering parameters ( $\tilde{e}_{ij}(s)$ ,  $i, j=1, 2$ ) is already built into the procedure. It should be noted that in choosing the polynomial  $h(s)$  and integer  $k$ ,  $h(0)=0$  and  $k=0$  cannot be allowed simultaneously, since this violates the lossless criterion of the equation (8).

Second, after the scattering parameters of the lossless matching network  $M$  have been obtained, the corresponding ones of the lossy matching network  $N$  can be constructed by the transformation of the Corollary.

Third, the definition of the TPG defined by authors is given by (refering to Fig. 1)

$$T(\omega) = \frac{\text{Power delivered to the load } Z_L}{\text{Power available from the generator}} = \prod_{k=1}^{N+1} T_k \quad (9)$$

where

$$T_k = \frac{\text{Power available from the FET}_k}{\text{Power available from the FET}_{k-1}} = \frac{(1 - |\Gamma_{g,k}|^2) |e_{21,k}|^2 |s_{21,k}|^2}{|1 - \Gamma_{g,k} e_{11,k}|^2 |1 - \Gamma_k s_{11,k}|^2 (1 - |\Gamma_{g,k+1}|^2)} \quad (10)$$

in which

$\tilde{e}_{11,k}$  is the number of the FETs;  
 $\Gamma_{g,k}$  is real normalized reflection coefficient of the equivalent Thevenin generator at the output of the FET<sub>k-1</sub>, and  
 $\Gamma_k$  is real normalized reflection coefficient at the output of the  $N_k$ .

$$\Gamma_k = e_{22,k} + \frac{e_{12,k} e_{21,k} \Gamma_{g,k}}{1 - e_{11,k} \Gamma_{g,k}} \quad (11)$$

$\Gamma_{g,k+1}$  is real normalized reflection coefficient similar to  $\Gamma_{g,k}$ .

$$\Gamma_{g,k+1} = s_{22,k} + \frac{s_{12,k} s_{21,k} \Gamma_k}{1 - s_{11,k} \Gamma_k} \quad (12)$$

$e_{ij,k}$  ( $i, j=1, 2$ ) is the unit normalized scattering parameters of  $N_k$ ; and  
 $s_{ij,k}$  ( $i, j=1, 2$ ) is the unit normalized scattering parameters of FET<sub>k</sub>.

In our example, a two-stage monolithic microwave integrated broadband amplifier is designed. The lossy element models in equations (2a) and (2b) are employed in the design of the matching networks of the amplifier. Inputs to the program developed by authors and the performance of the amplifier are summarized as follows.

Generator:  $Z_g = 50 \Omega$

Load:  $Z_L = 50 \Omega$

Skin loss of the inductor:  $Q_L = 25$  or  $d_L = 11863.53$

Skin loss of the capacitor:  $Q_c = 50$  or  $d_c = 1.30446 \times 10^{10}$

Dielectric loss of the capacitor:  $Q_d = 100$

Frequency at which the above losses are measured  $f_o = 14 \text{GHz}$ ;

Passband:  $7 \text{GHz} \leq f \leq 14 \text{GHz}$

Maximum complexity of the matching networks

input matching network:  $n=4, k=1$

interstage matching network:  $n=4, k=1$

output matching network:  $n=3, k=1$

Flat gain level to be approximated

$T_{o1}(\omega) = 4.77 \text{dB}$

$T_{o2}(\omega) = 9.54 \text{dB}$

Scattering parameters of FETs are the same as that of REF.(1).

Result of optimization:

$$\tilde{e}_{11,1}(s) = \frac{-1.014 - 0.8305s - 1.332s^2 - 1.804s^3}{1.014 + 1.9810s + 2.434s^2 + 1.804s^3} \quad (13a)$$

$$\tilde{e}_{11,2}(s) = \frac{-0.9452 + 0.2347s + 2.240s^2 + 0.9348s^3 + 5.9s^4}{0.9452 + 4.1910s + 6.495s^2 + 7.1460s^3 + 5.9s^4} \quad (13b)$$

$$\tilde{e}_{11,3}(s) = \frac{-0.6746 + 0.8165s - 0.6979s^2 + 2.03s^3}{0.6746 + 1.9370s + 2.2440s^2 + 2.03s^3} \quad (13c)$$

The two-stage amplifier is shown in Fig.(2) and its performance is given by

$$T(\omega) = 8.905 \pm 1.235 \text{ dB}$$

which is shown in Fig.(3)

The advantages of the "Real Frequency" technique is clearly shown in the example. The program can automatically find optimum topologies for the lossy matching networks within the maximum degree of the denominator  $n$ , when the  $k$  is fixed, (e.g., the actual degree of the denominator of the equation (13a) is 3 instead of 4 specified). The synthesis of the matching networks is carried out along with the numerical work by continuous fraction expansion of the input impedances which correspond to the equations (13a), (13b) and (13c). The final input bandpass matching network with four elements, in which the ideal transformer is canceled, can be obtained by the well-known Norton transformation, so do the inter-stage and the output bandpass matching networks with five and four elements.

**Discussion:** This example is similar to that of Ref.(1), but the computational steps are quite simplified. Our method can not only synthesize the lossy matching networks directly, but also employ any complex models of lumped elements with arbitrary nonuniform losses, so that the performance of the amplifier can approach the actual one very well from theoretical point of view.

#### ACKNOWLEDGMENT

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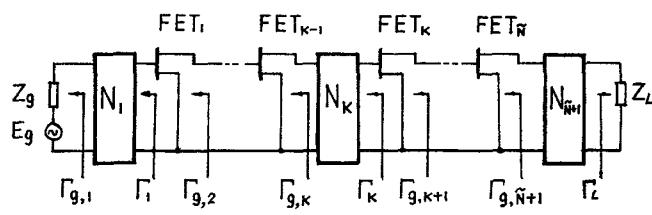


Fig. 1 The multistage FET amplifier used for defining the TPG

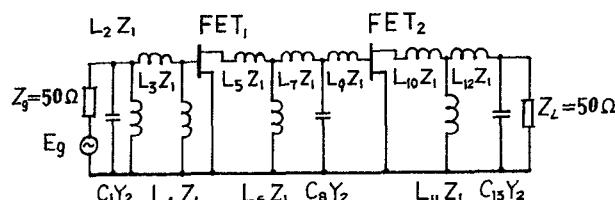


Fig. 2 Design of two-stage amplifier  $C_1=7442 \text{ pF}$ ,  $L_2=3458 \text{ nH}$ ,  $L_3=6262 \text{ nH}$ ,  $L_4=4.1564 \text{ nH}$ ,  $L_5=1.0298 \text{ nH}$ ,  $L_6=3015 \text{ nH}$ ,  $L_7=0.0603 \text{ nH}$ ,  $C_8=8052 \text{ pF}$ ,  $L_9=8283 \text{ nH}$ ,  $L_{10}=0.739 \text{ nH}$ ,  $L_{11}=4214 \text{ nH}$ ,  $L_{12}=0.0509 \text{ nH}$ ,  $C_{13}=5967 \text{ pF}$ ,  $Z_1=(j\omega+11863.53\omega^{1/2})$ ,  $Y_2=1/(1/j\omega+\omega^{1/2}1.30446 \times 10^{16} + 1/100\omega)$

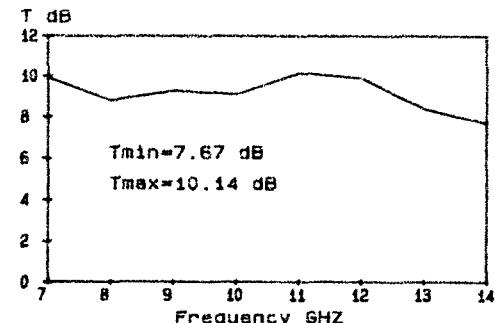


Fig.3 Performance of the amplifier shown in Fig.2